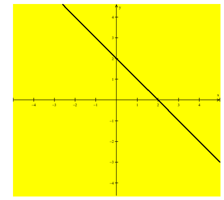
How to determine the portion and direction of a parametric curve when you are able to eliminate the parameter

Suppose you're given the parametric equations $\begin{aligned} x &= 2+t^2 \\ y &= -t^2 \end{aligned}$



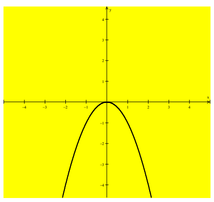
which corresponds to the rectangular equation $y = 2 - x$ (shown on the right ➡),

and you need to determine which part of the graph is being traced out and in what orientation.

1. Decide which parametric equation ($x =$ or $y =$) you find easier to analyze.

   $y = -t^2$ will be easier

2. Sketch a graph of the function you chose in step 1, with the horizontal axis representing t, and the vertical axis representing whichever variable you chose in step 1 (in this case, y).

3. Describe what's happening on the graph in step 2 as t goes from $-\infty$ to ∞ (ie. as you move from left to right). Every time the graph changes general direction (from increasing to decreasing, from decreasing to increasing, or making a sudden discontinuous jump), describe that change (from what value to what value).

   As t goes from $-\infty$ to ∞,
   $y = -t^2$ increases from $-\infty$ to 0, then decreases to $-\infty$

4. Go to the graph of the original rectangular equation and identify which "points" on its graph correspond to the $x-$ or $y-$values you found in step 3.

   Remember that
   $x = -\infty$ corresponds to the far left side of the graph | $y = -\infty$ corresponds to the bottom of the graph
   $x = \infty$ corresponds to the far right side of the graph | $y = \infty$ corresponds to the top of the graph
   $x = 0$ corresponds to the $y-$intercept of the graph | $y = 0$ corresponds to the $x-$intercept of the graph
   you can use the rectangular equation to find specific points

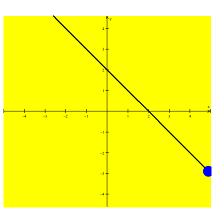
   and
   x increasing corresponds to moving right | y increasing corresponds to moving up
   x decreasing corresponds to moving left | y decreasing corresponds to moving down

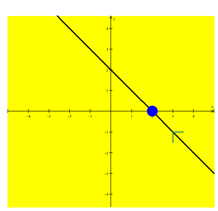
   $y = -t^2$ goes from $-\infty$ to 0 to $-\infty$, so the parametric curve goes from

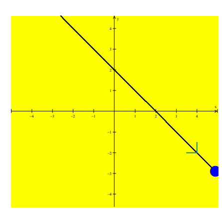
   the bottom of the graph of $y = 2 - x$

   up to the $x-$intercept $(2, 0)$

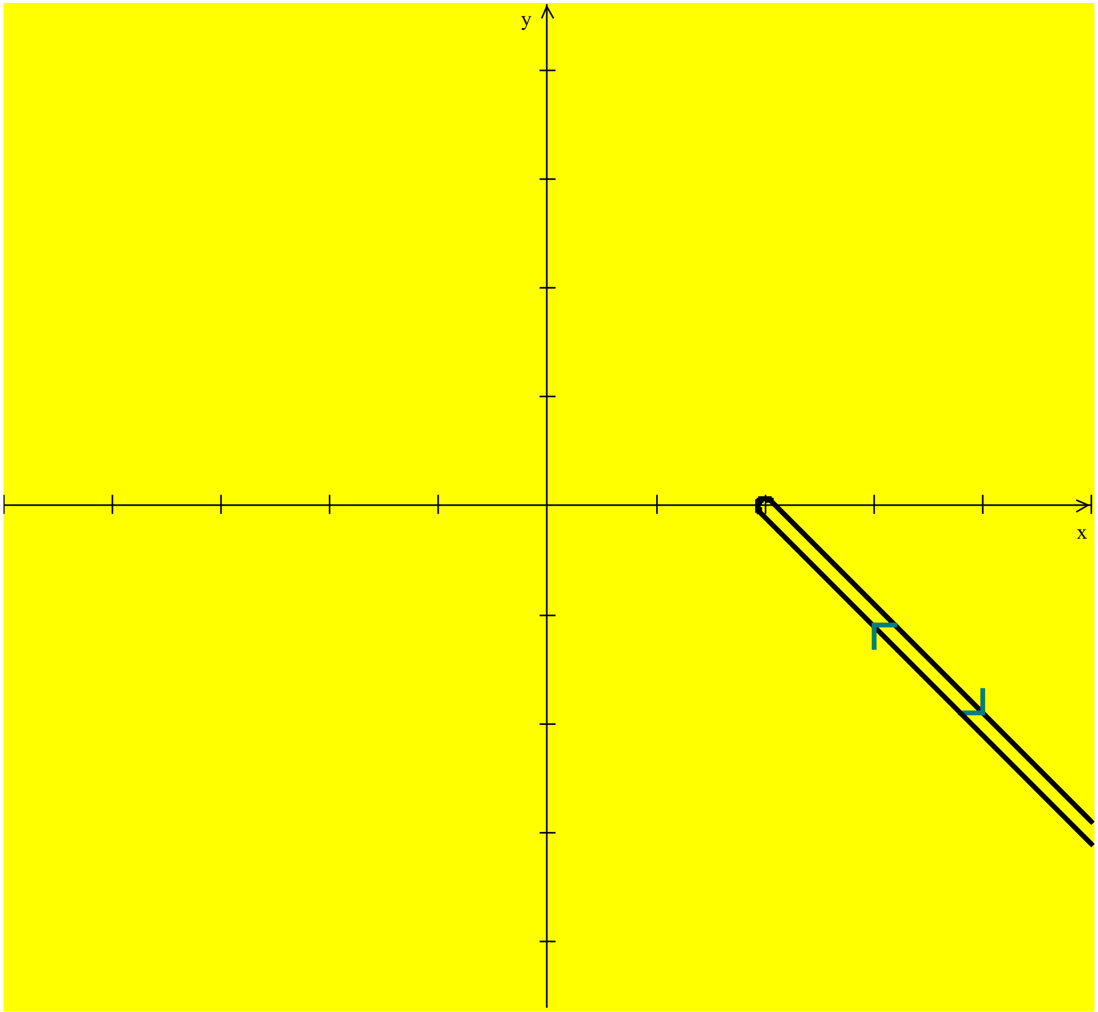
   down to the bottom

5. Sketch out only what you described in step 4.



YOUR TURN: **(check using your calculator AFTER you have a solution)**

A. Analyze the parametric equations $\begin{array}{l} x=-t^4 \\ y=-t^8 \end{array}$, which correspond to the rectangular equation $y=-x^2$, by analyzing the $x=$ equation.

B. Analyze the parametric equations $\begin{array}{l} x=e^{-t} \\ y=2-e^{-t} \end{array}$, which correspond to the rectangular equation $y=2-x$, by analyzing the $x=$ equation.

C. Analyze the parametric equations $\begin{array}{l} x=e^{2t} \\ y=-e^{t} \end{array}$, which correspond to the rectangular equation $x=y^2$.
You must decide which parametric equation to analyze (try both, but one will be easier than the other).

D. Analyze the parametric equations $\begin{array}{l} x=\sin t \\ y=\cos^2 t \end{array}$, which correspond to the rectangular equation $y=1-x^2$.
You must decide which parametric equation to analyze.

E. Analyze the parametric equations $\begin{array}{l} x=4-2\ln t \\ y=\ln t \end{array}$.
You must find the rectangular equation, and decide which parametric equation to analyze.